Susceptible-Infected- Removed Models when Agents Respond to Different Information Sets

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Abstract

The properties of the models of epidemiology are well known. Here, we consider the standard SIR model of epidemiology extended to incorporate behavioural responses by agents.

Introducing such endogenous behaviour by Susceptibles into a standard SIR model is known to influence the solution paths of the models in ways which are highly relevant to current policy debates over the release of lockdown [for example, 1].

Here, we examine two variants of this approach. In one, agents modify their behaviour homogenously as the proportion of Infecteds rise. This reduces the proportion of the population which over time experience the disease. In addition, the peak infection rate is reduced very substantially.

In the second, there are two groups with different death rates to the disease, one of which is very low. We assume that the agents extend the set of information to which they respond. In addition to the infection rate, they take into account the death rates of their respective groups. These assumptions give an outcome which is much closer to the basic SIR model results.

1. Introduction

The properties of the basic Susceptible-Infected-Recovered model [2-4] are well known.

Currently, many countries are experiencing lockdown as a strategy to deal with Covid19. Epidemiological models based on the principles of SIR models predict a second large peak of infections if the lockdowns are lifted [5, for example, is a paper which has had considerable influence on policy].

If behaviour reverts to that which obtained before the crisis, these predictions will be correct. However, the central insight of economics is that agents alter their behaviour when the set of incentives which they face alters.

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Agents have experienced a serious health crisis, the worst in most Western countries since the Spanish flu 100 years ago.

We examine the implications of agents using information on the spread or consequences of the virus to modify their behaviour.

First, we set out an aggregate SIR model in which the Susceptibles modify their behaviour with respect to the proportion of Infecteds in the population. We then examine an SIR model with two groups with very different death rates and examine the consequences if agents modify behaviour in the light of not the infection rate but the death rates in their respective groups.

In other words, agents in the two variants respond to different sets of information.

2. The models

Model A

We make the simplifying assumption that individuals have correct information on the proportion of the population which is infected at any given time. This could readily be extended to encompass the concept of imperfect information, for example, in its various guises. Our purpose here, however, is to illustrate the effect of agents changing their behaviour with respect to different sets of information.

We assume that Susceptibles alter behaviour as the proportion of Infected increases in a way which essentially reduces the value of the parameter β.

Specifically, we have:

$$\beta = (1 - \delta I)\beta_0 \geq 0$$

$$\frac{dS}{dt} = -(1 - \delta I)\beta_0 SI$$

$$\frac{dI}{dt} = (1 - \delta I)\beta_0 SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Where S, I and R are the proportions of Susceptibles, Infected and Removed in the population ($S + I + R = 1$), β is the probability of transmission, γ is the recovery rate and δ measures the strength of the behavioural response.

Model B

Here there are two groups in the population and they alter behaviour with respect to both the overall infection rate and the observed death rates in their own group:

$$\beta_1 = (1 - \alpha_1 \eta_1 I)\beta_0 \geq 0$$
$$\beta_2 = (1 - \alpha_2 \eta_2 I)\beta_0 \geq 0$$

$$\frac{dS_1}{dt} = -(1 - \alpha_1 \eta_1 I)\beta_0 S_1 I$$
$$\frac{dS_2}{dt} = -(1 - \alpha_2 \eta_2 I)\beta_0 S_2 I$$

$$\frac{dI}{dt} = [(1 - \alpha_1 \eta_1 I)\beta_0 S_1 + (1 - \alpha_2 \eta_2 I)\beta_0 S_2]I \; - \gamma I$$

$$\frac{dD_1}{dt} = -\eta_1 \frac{dS_1}{dt} = \eta_1 (1 - \alpha_1 \eta_1 I)\beta_0 S_1 I$$
$$\frac{dD_2}{dt} = -\eta_2 \frac{dS_2}{dt} = \eta_2 (1 - \alpha_2 \eta_2 I)\beta_0 S_2 I$$

$$\frac{dR}{dt} = \gamma I$$

Where α_i is the response parameter for group *i* with respect to the death rate η_i and D_i is the deaths for group *i*.

3. Results

We solve the systems using the 'ode' function in the 'deSolve' R package (we based it on the code here http://rstudio-pubs-static.s3.amazonaws.com/6852_c59c5a2e8ea3456abbeb017185de603e.html)[4]

[4] The R code is available from Nyman rickardnyman@gmail.com

In the Covid crisis, the reproduction number is thought to be between 2 and 3.5 [for example, 5]. We use for illustration a value of 2.5. The initial conditions are S = 0.999999, I = 0.000001, R =0. We assume β = 0.25 and $\gamma = 0.1$.

Figure 1a shows the solution of the standard SIR model in which δ = 0. In other words, there is no behavioural response.

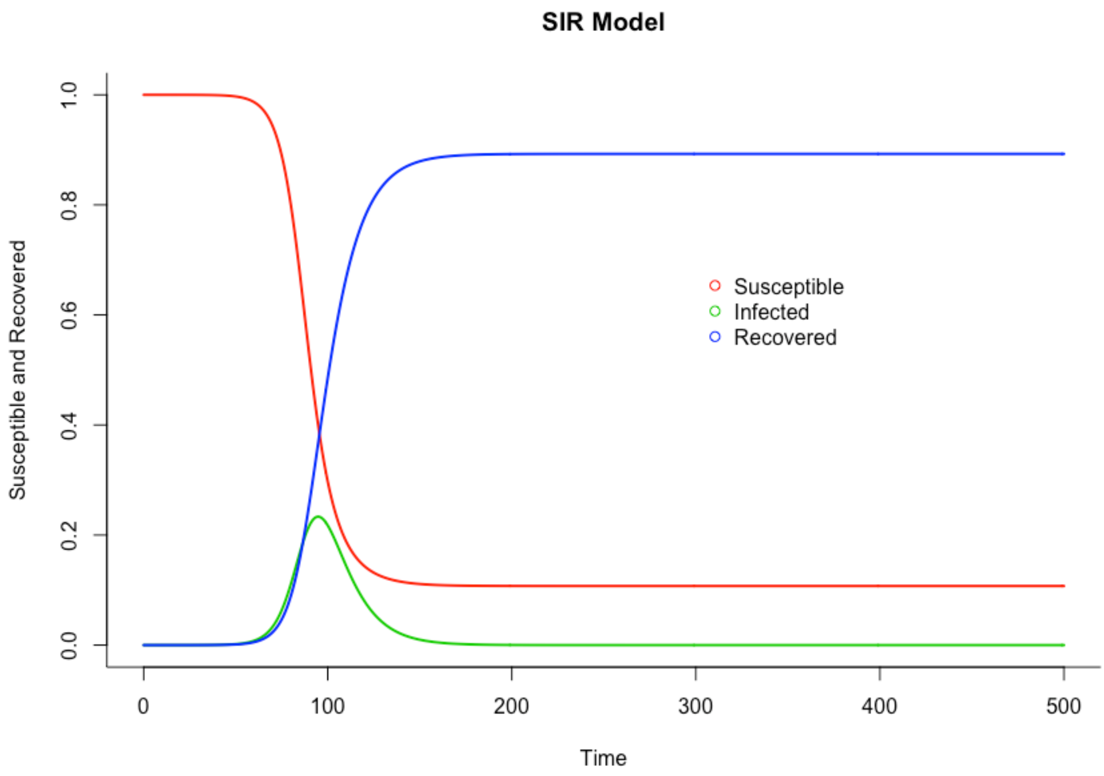

Figure 1a *Standard SIR model, S(0) = 0.999999, I(0) = 0.000001, β = 0.25, $\gamma = 0.1$, δ = 0*

The peak value of I is 0.233, which occurs in step 96 of the solution. The converged value of R, in other words the proportion of the population which experience the virus, is 0.893.

Figure 1b illustrates a strong endogenous response in which δ = 8.

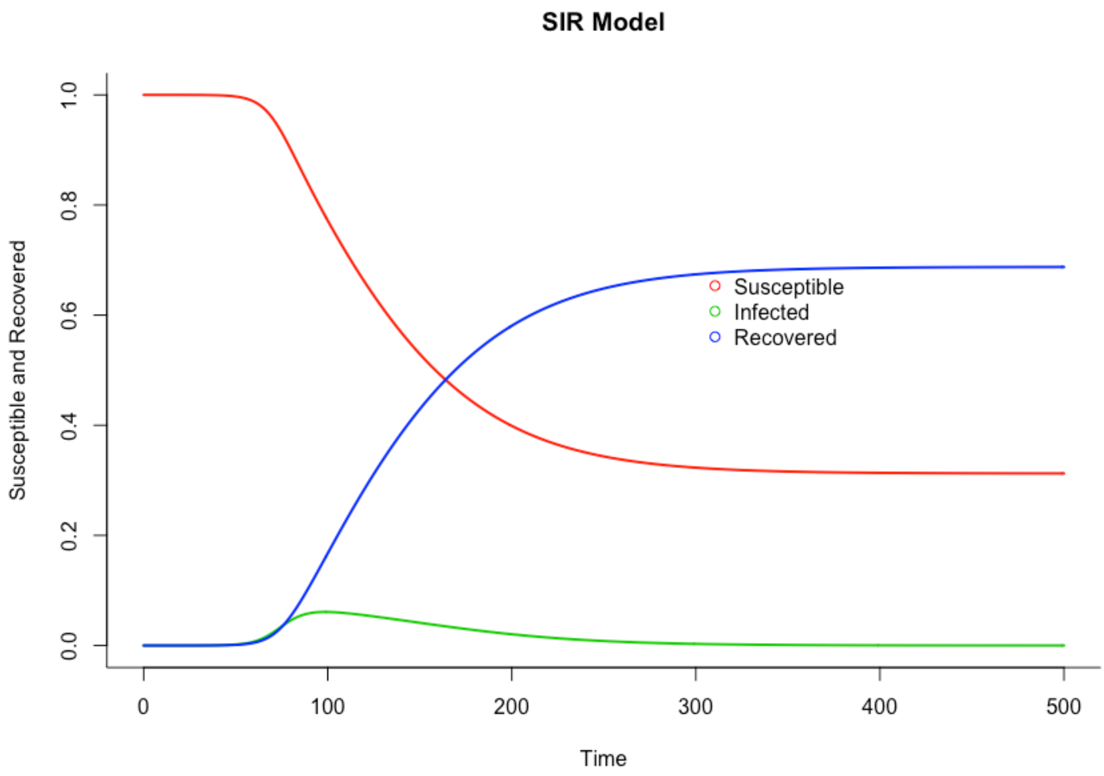

Figure 1b *SIR model with behavioural response with respect to I, S(0) = 0.999999, I(0) = 0.000001, β = 0.25, γ = 0.1 , δ = 8.*

The peak value of I is 0.061, which occurs in step 100 of the solution. The converged value of R, in other words the proportion of the population which experience the virus, is 0.688.

We present in Figure 2 an illustrative solution of the SIR model with two groups, with respective death rates of 0.1 per cent and 5 per cent. We assume here that agents modify behaviour with respect to the information on the respective death rates in addition to infection rates.

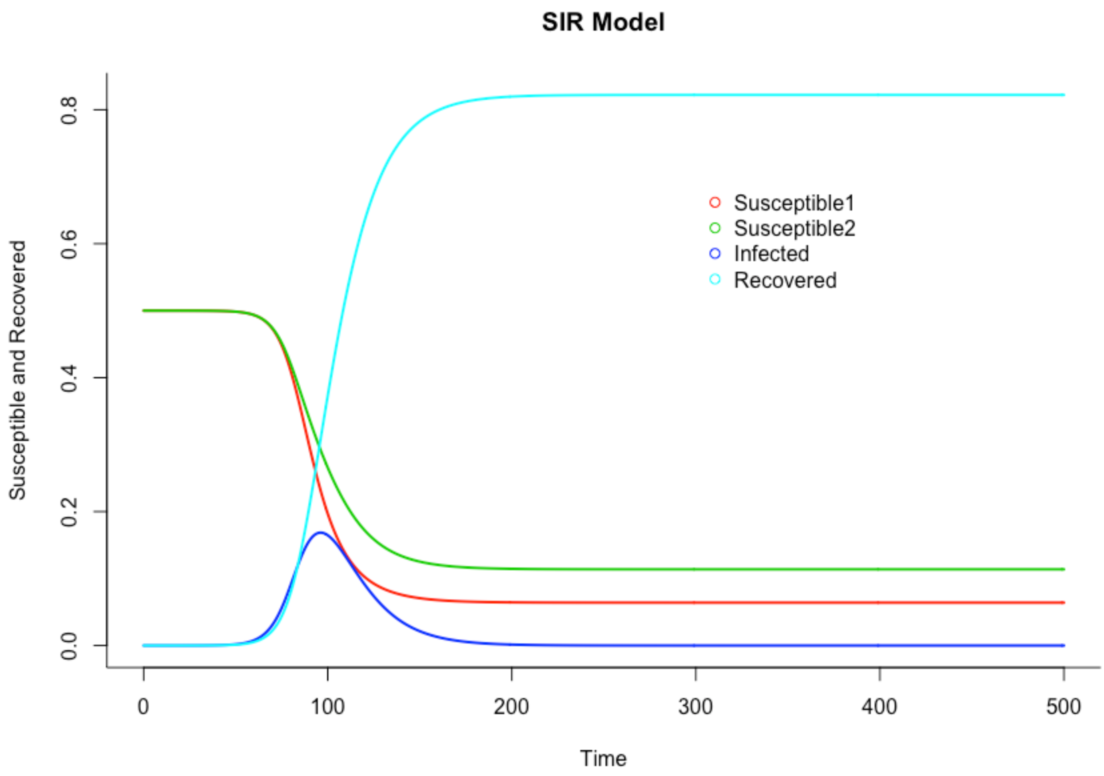

Figure 2 *SIR model with two groups and with behavioural response with respect to the death rate of each group,* $S_1(0) = 0.4999995, S_2(0) = 0.4999995$*, I(0) = 0.000001,* $\beta = 0.25, \gamma = 0.1$ *,* $\alpha_1 = 1$*,* $\alpha_2 = 50$*,* $\eta_1 = 0.001$*,* $\eta_2 = 0.05$

We set the relative values of the α_i in proportion to assumed death rates in the two groups. If we set them equal to each other, the response would be linear. The groups respond entirely in proportion to their relative death rate. We assume here a non-linear response, with those in the lower mortality group under-reacting in response to the death rate, relative to those in the higher mortality group.

The peak value of I is 0.169, which occurs in step 97 of the solution. The converged value of R, in other words the proportion of the population which experience the virus, is 0.822. These results are very similar to those in the basic SIR model. The peak infection rate and the proportion eventually infected are lower, but the difference is small.

4. Brief discussion

In the current crisis, most countries have imposed some form of lockdown. A key policy question is how to lift them. A major problem facing policy makers is that pure epidemiological models will show that a second, large wave of infection takes place.

However, this could be mitigated to a very considerable extent if agents react to incentives and do not revert to pre-virus behaviour.

If agents use the information contained in infection rates and modify behaviour with respect to this, the proportion of the population which eventually gets the virus is reduced substantially. Perhaps more importantly, peak infection rate is reduced to around one third of its value in a model with no behavioural response.

However, with two groups with markedly different death rates and when each group modifies behaviour with respect to both its own death rate and the infection rate, the outcome is more similar to, though still different from, the basic SIR model.

The policy implication is that a relaxation of lockdowns will not necessarily lead to a second large peak of infections which epidemic models without behavioural response imply. However, the information set which agents use when considering modification of behaviour is crucial.

References

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